FETI and HFETI Solvers

- C++ implementation based on Intel MKL sparse and dense BLAS routines and MKL version of PARASOL sparse direct solver
- Parallelization tools and strategies:
  - Hybrid parallelization for multisocket, multicore, non-shared memory environments
  - Distributed memory parallelization - use of MPI 3.0 non-blocking collective operations for global reductions - Intel MPI 5.0
  - Number of subdomains and MPI process iterators only overlap if:
    - Shared memory parallelization using Intel Cilk+
    - Enables parallel reduction using custom reduce operators on C++ classes

Reduction of global communication for (H)FETI

- Each MPI process iterates only over lambda bars associated with given subdomain
- FETI - A required for multiplication with the restriction of B matrix to given subdomain
- HFETI - A required for multiplication with the restriction of B1 matrix to given subdomain
- Global update of vector λ becomes only nearest the neighbor type of communication with good scalability

Hiding latencies in Krylov Solvers

Preconditioned pipelined Conjugate Gradient (CG) algorithm

1. \( r_i = b - A_i x_i \)
2. \( p_i = r_i \)
3. \( d_i = B_i r_i \)
4. \( A_i p_i = d_i \implies d_i \) est. product - global communication - scales as \( \log P \)
5. \( r_{i+1} = r_i - \lambda d_i \)
6. if \( \lambda > 0 \): then
   - \( \lambda_{i+1} = \sqrt{1 + \frac{r_{i+1}^2}{r_i^2}} \)
   - \( r_{i+1} = \frac{r_i}{\lambda_{i+1}} - r_{i+1} \)
7. else:
   - \( r_{i+1} = \lambda_{i+1} r_i \)
8. end if
9. add \( \lambda_{i+1} r_i \) to \( r_{i+1} \)
10. end if

Superlinear Strong Scaling of FETI

- 2.5 millions of DOFs using FETI solver
- Decomposed into 34, 64, 128, 256, 512 and 1024 subdomains
- Measured on Anselm supercomputer
  - CG nodes with 2 64Gbit/s (164Gbit/s) subdomains per node

Benchmark Systems

- IT4Innovations - www.it4i.cz: Anselm - up to ~3200 cores
  - own blocking cluster of 2700 nodes each with
    - 16 cores Intel Sandy Bridge E5-2697 2.70GHz 155W 64/128-bit
    - Intel-IBM DNN network - 64 Dip's inter-node bandwidth
- SURFsara e+ - Cartesius - up to ~8000 cores
  - own blocking cluster of 3600 nodes each with
    - 16 cores Intel Xeon E5-2670 2.60GHz 105W 64/128-bit
    - Intel-IBM DNN network - 56 Dip's inter-node bandwidth

Intercluster Processing

Cluster size: ~320 000 DOFs; optimal decomposition into 27 subdomains

<table>
<thead>
<tr>
<th>Domain size</th>
<th>Subdomains</th>
<th>Optimal</th>
<th>Iterations</th>
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</thead>
<tbody>
<tr>
<td>27</td>
<td>3</td>
<td>27</td>
<td>55</td>
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<tr>
<td>64</td>
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<td>64</td>
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FETI and HFETI per iteration time

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HFETI vs FETI - one iteration time (measured on Anselm)

- hfetei: 45.55s
- feti: 70.38s

HFETI vs FETI - one iteration time (measured on Cartesius)

- hfetei: 43.55s
- feti: 69.38s

HFETI vs FETI - one iteration time (measured on Dnieper)

- hfetei: 41.55s
- feti: 67.38s

Total FETI - Large scale benchmarks

- Preconditioning, number of iterations and solution time

<table>
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<th>Solution time</th>
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<td>2000000 DOFs</td>
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<tr>
<td>5000000 DOFs</td>
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<td></td>
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<tr>
<td>10000000 DOFs</td>
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