Space-filling Curves for Domain Decomposition in Scientific Simulations

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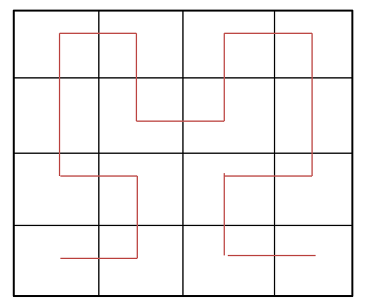
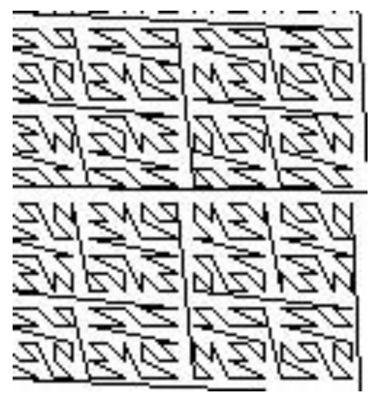
What are Space-filling Curves(SFC)?

- A space-filling curve F:R->R^d provides a mapping from points on the curve to points in d-dimensional space.
- F^{-1} can be used to create a locality preserving order of points in higher dimensions.
- The curve can be sliced into p pieces to create p partitions.
- Example : Hilbert Curve[1] defined on 16 equi-distant points on a plane



Why are we interested in Space-filling Curves?

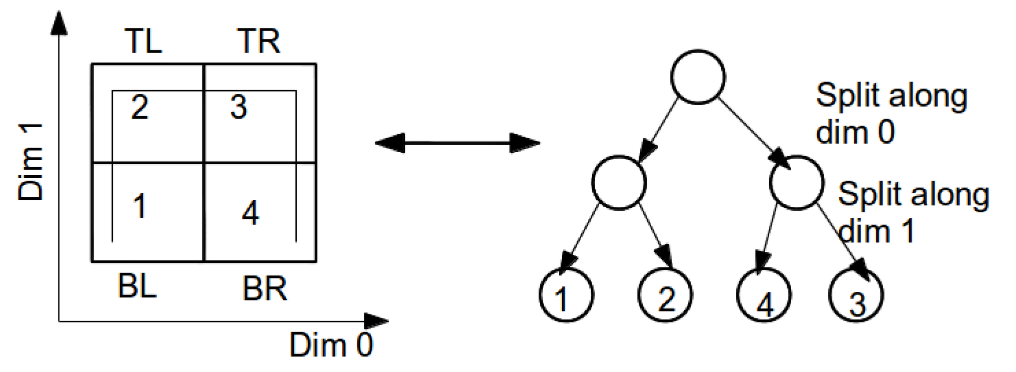
- Less space and time overheads.
- Existing algorithms are dependent on the shape and size of the domain. Eg: Hilbert curve can only be generated for symmetric domains with dimension a power of 2.
- Morton order(Z-order) is a general technique, has poor locality :



- Therefore, there is need for a general SFC with good locality that can be used to partition meshes of arbitrary shapes and sizes.

General SFC Algorithm

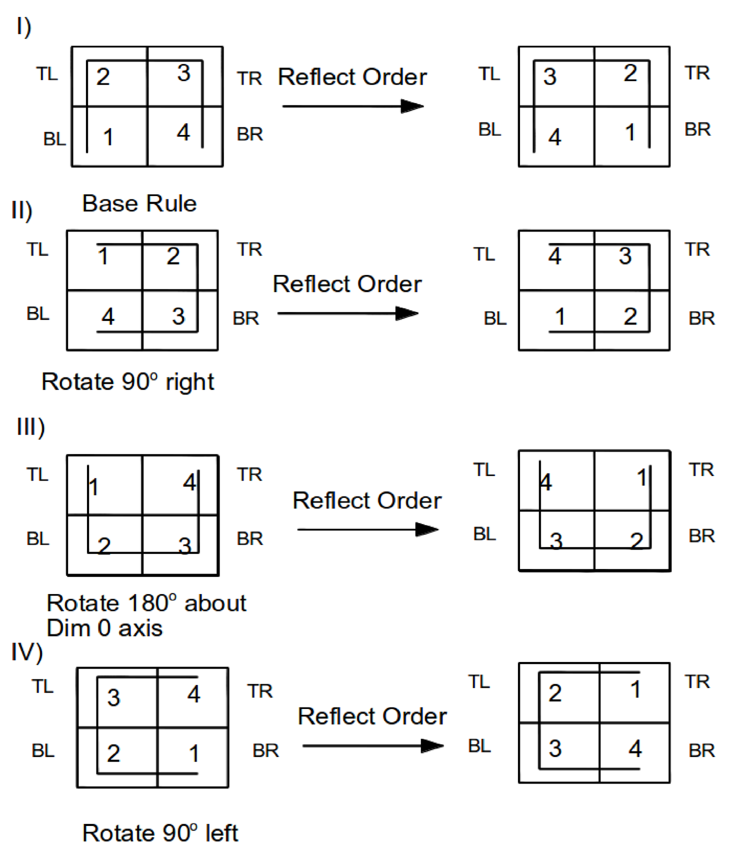
- We use recursion to generate the space-filling curve
- We first build a kd-tree out of the points in the domain. Each node of the kd-tree corresponds to a subdomain.
- The tree is then traversed top to bottom based on a set of rules. The order of traversal of the kd-tree generates the SFC.



- Partially refined domains are treated as special cases to avoid large discontinuities in the generated curve

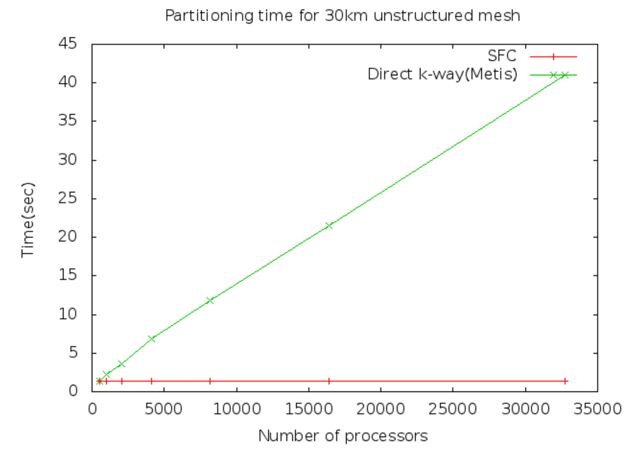
2D SFC Rules

- Bounding box is a four-sided polygon. So we have used four directions and entry/exit points to define the rules
  - Directions : top, bottom, left, right
  - Entry/exit points : top-left, top-right, bottom-left, bottom-right



Applications and Results

- For the 2D SFC, we used meshes from the Community Earth System Model (CESM) as test cases. They include : structured, block structured adaptive, unstructured and unstructured adaptive
- Partitioning time of the SFC algorithm vs the direct k-way algorithm of Metis. E.g: Partitioning time for the 30km unstructured atmosphere mesh



- Compared the quality of partitions with respect to their maximum volume (computational load) and maximum surface area(communication load)
- Quality of partitions generated by the SFC for a structured 2D atmosphere mesh with 1152 longitudes and 768 latitudes.

	Direct K-way		SFC	
#procs	Max Load	Max Surf Area	Max Load	Max Surf Area
512	1751	258	1728	188
1000	905	207	885	202
1024	889	198	864	117
2048	444	134	432	92
4096	222	91	216	56
8192	111	64	108	44

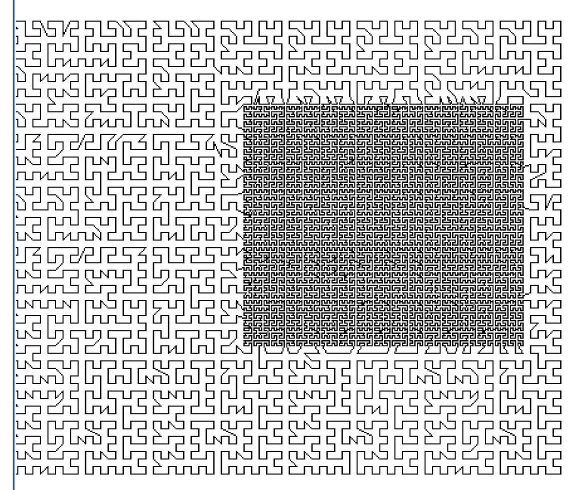
Applications and Results (contd.)

- Quality of partitions for an unstructured atmosphere mesh from the MPAS model.

	Direct k-way		SFC	
#procs	Max Load	Max Surf Area	Max Load	Max Surf Area
512	1308	172	1281	174
1024	659	143	641	124
2048	329	88	321	90
4096	164	61	161	62
8192	82	39	81	43

SFC partition of the northern hemisphere and comparison with Metis for the 30km unstructured MPAS mesh.

- Quality of partitions for a block structured adaptive mesh from WRF.



	Z-order		SFC	
#procs	Max Load	Max Surf	Max Load	Max Surf
8	1765	241	1765	218
16	882	185	882	172
40	353	136	353	124
50	282	116	282	98
64	220	98	220	96
80	176	90	176	85

SFC partition of 61X74 WRF grid with one nested domain of size 97X112 and comparison with Z-order

Conclusions & Future Work

- SFCs seem to be a good option to generate good quality partitions quickly and with less overhead
- We are currently working on further improving the quality of partitions using refined performance models of the applications